

# New sizes of complete arcs in $PG(2, q)$

ALEXANDER A. DAVYDOV

adav@iitp.ru

Institute for Information Transmission Problems, Russian Academy of Sciences,

Bol'shoi Karetnyi per. 19, GSP-4, Moscow, 127994, Russia

GIORGIO FAINA

faina@dipmat.unipg.it

STEFANO MARCUGINI

gino@dipmat.unipg.it

FERNANDA PAMBIANCO

fernanda@dipmat.unipg.it

Dipartimento di Matematica e Informatica, Università degli Studi di Perugia, Via  
Vanvitelli 1, Perugia, 06123, Italy

**Abstract.** New upper bounds on the smallest size  $t_2(2, q)$  of a complete arc in the projective plane  $PG(2, q)$  are obtained for  $853 \leq q \leq 2879$  and  $q = 3511, 4096, 4523, 5003, 5347, 5641, 5843, 6011$ . For  $q \leq 2377$  and  $q = 2401, 2417, 2437$ , the relation  $t_2(2, q) < 4.5\sqrt{q}$  holds. The bounds are obtained by finding of new small complete arcs with the help of computer search using randomized greedy algorithms. Also new sizes of complete arcs are presented.

## 1 Introduction

Let  $PG(2, q)$  be the projective plane over the Galois field  $F_q$ . An  $n$ -arc is a set of  $n$  points no 3 of which are collinear. An  $n$ -arc is called complete if it is not contained in an  $(n + 1)$ -arc of  $PG(2, q)$ . Surveys of results on arcs can be found in [9, 10]. In [10] the close relationship between the theory of complete  $n$ -arcs, coding theory and mathematical statistics is presented. In particular a complete arc in a plane  $PG(2, q)$ , points of which are treated as 3-dimensional  $q$ -ary columns, defines a parity check matrix of a  $q$ -ary linear code with codimension 3, Hamming distance 4 and covering radius 2. Arcs can be interpreted as linear maximum distance separable (MDS) codes and they are related to optimal coverings arrays [8] and to superregular matrices [11].

One of the main problems in the study of projective planes, which is also of interest in Coding Theory, is the determination of the spectrum of possible sizes of complete arcs. Especially the problem of determining  $t_2(2, q)$ , the size of the smallest complete arc in  $PG(2, q)$ , is interesting.

In Section 2 we give upper bounds on  $t_2(2, q)$  for  $853 \leq q \leq 2879$  and  $q = 3511, 4096, 4523, 5003, 5347, 5641, 5843, 6011$ . These bounds are new for almost all  $q$ . For  $q \leq 2377$  and  $q = 2401, 2417, 2437$ , the relation  $t_2(2, q) < 4.5\sqrt{q}$  holds. For smaller  $q$  slightly smaller bounds hold. The upper bounds have been obtained by finding of new small complete arcs with the help of the randomized greedy algorithms described in [1, Sect. 2], [5, Sect. 2].

In Section 3 we present new sizes of complete arcs in  $PG(2, q)$  with  $169 \leq q \leq 349$  and  $q = 1013, 2003$ .

## 2 Small complete $k$ -arcs in $PG(2, q)$ , $853 \leq q \leq 2879$

In the plane  $PG(2, q)$ , we denote  $\bar{t}_2(2, q)$  the smallest *known* size of complete arcs. For  $q \leq 841$ , the values of  $\bar{t}_2(2, q) < 4\sqrt{q}$  are collected in [2, Tab. 1].

In Tables 1 and 2, the values of  $\bar{t}_2(2, q)$  for  $853 \leq q \leq 2879$  and  $q = 3511, 4096, 4523, 5003, 5347, 5641, 5843, 6011$  are given. We denote  $A_q = \lfloor 4.5\sqrt{q} - \bar{t}_2(2, q) \rfloor$ ,  $B_q$  a superior approximation of  $\bar{t}_2(2, q)/\sqrt{q}$ . Also,  $C_q = \lfloor 5\sqrt{q} - \bar{t}_2(2, q) \rfloor$ . For all  $q$  in Table 1 and  $q = 2401, 2417, 2437$  in Table 2, it holds that  $\bar{t}_2(2, q) < 4.5\sqrt{q}$ .

In [7], complete  $k$ -arcs are obtained with  $k = 4(\sqrt{q}-1)$ ,  $q = p^2$  odd,  $q \leq 1681$  or  $q = 2401$ . For even  $q = 2^h$ ,  $10 \leq h \leq 15$ , the smallest known sizes of complete  $k$ -arcs in  $PG(2, q)$  are obtained in [3], see also [2, p. 35]. They are as follows:  $k = 124, 201, 307, 461, 665, 993$ , for  $q = 2^{10}, 2^{11}, 2^{12}, 2^{13}, 2^{14}, 2^{15}$ , respectively. Also,  $6(\sqrt{q}-1)$ -arcs in  $PG(2, q)$ ,  $q = 4^{2h+1}$ , are constructed in [4]; for  $h \leq 4$  it is proved that they are complete. It gives a complete 3066-arc in  $PG(2, 2^{18})$ . In Tables 1 and 2, we use the results of [3, 7] for  $q = 31^2, 37^2, 41^2, 7^4, 2^{10}, 2^{11}$ .

The rest of sizes  $k$  for small complete arcs in Tables 1 and 2 is obtained in this work by computer search with the help of the randomized greedy algorithms.

From Tables 1 and 2, we obtain Theorems 1 and 2.

**Theorem 1.** In  $PG(2, q)$ ,

$$\begin{aligned} t_2(2, q) &< 4.5\sqrt{q} \quad \text{for } q \leq 2377, q = 2401, 2417, 2437; \\ t_2(2, q) &< 4.2\sqrt{q} \quad \text{for } q \leq 1163, q = 1181, 1193, 1369, 1681, 2401; \\ t_2(2, q) &< 4.3\sqrt{q} \quad \text{for } q \leq 1451, q = 1459, 1471, 1481, 1483, 1493, 1499, 1511, \\ &\quad 1681, 2401; \\ t_2(2, q) &< 4.4\sqrt{q} \quad \text{for } q \leq 1849, q = 1867, 1889, 1901, 1907, 1913, 1949, 1993, \\ &\quad 2401. \end{aligned}$$

**Theorem 2.** In  $PG(2, q)$ ,

$$\begin{aligned} t_2(2, q) &< 4.5\sqrt{q} \quad \text{for } q \leq 2377, q = 2401, 2417, 2437; \\ t_2(2, q) &< 4.5\sqrt{q} - 10 \quad \text{for } q \leq 1163, q = 1181, 1187, 1193, 1223, 1237, 1249, \\ &\quad 1369, 1681, 2401; \\ t_2(2, q) &< 4.5\sqrt{q} - 8 \quad \text{for } q \leq 1423, q = 1429, 1433, 1439, 1447, 1451, 1471, \\ &\quad 1481, 1483, 1499, 1511, 1681, 2401; \\ t_2(2, q) &< 4.5\sqrt{q} - 6 \quad \text{for } q \leq 1693, q = 1699, 1709, 1747, 1783, 2401; \\ t_2(2, q) &< 4.5\sqrt{q} - 3 \quad \text{for } q \leq 2003, q = 2017, 2027, 2401. \end{aligned}$$

Our methods allow us to obtain small arcs in  $PG(2, q)$  for  $q \leq 6011$ , using our present computers. We plan to write on these arcs sizes in a journal paper.

Let  $c$  be a constant independent of  $q$ . Let  $t(\mathcal{P}_q)$  be the size of the smallest complete arc in any projective plane  $\mathcal{P}_q$  of order  $q$ . In [12], for sufficiently large

Table 1. The smallest known sizes  $\bar{t}_2 = \bar{t}_2(2, q) < 4.5\sqrt{q}$  of complete arcs in planes  $PG(2, q)$ .  $A_q = \lfloor 4.5\sqrt{q} - \bar{t}_2(2, q) \rfloor$ ,  $B_q > \bar{t}_2(2, q)/\sqrt{q}$

$q$	$\bar{t}_2$	$A_q$	$B_q$	$q$	$\bar{t}_2$	$A_q$	$B_q$	$q$	$\bar{t}_2$	$A_q$	$B_q$
853	118	13	4.05	1087	137	11	4.16	1327	155	8	4.26
857	119	12	4.07	1091	138	10	4.18	1331	155	9	4.25
859	119	12	4.07	1093	138	10	4.18	1361	157	9	4.26
863	119	13	4.06	1097	138	11	4.17	1367	158	8	4.28
877	120	13	4.06	1103	138	11	4.16	1369	144	22	3.90
881	121	12	4.08	1109	138	11	4.15	1373	158	8	4.27
883	121	12	4.08	1117	140	10	4.19	1381	159	8	4.28
887	121	13	4.07	1123	139	11	4.15	1399	160	8	4.28
907	123	12	4.09	1129	140	11	4.17	1409	160	8	4.27
911	123	12	4.08	1151	142	10	4.19	1423	161	8	4.27
919	124	12	4.10	1153	142	10	4.19	1427	162	7	4.29
929	125	12	4.11	1163	143	10	4.20	1429	161	9	4.26
937	126	11	4.12	1171	144	9	4.21	1433	161	9	4.26
941	126	12	4.11	1181	144	10	4.20	1439	161	9	4.25
947	127	11	4.13	1187	145	10	4.21	1447	162	9	4.26
953	127	11	4.12	1193	145	10	4.20	1451	163	8	4.28
961	120	19	3.88	1201	146	9	4.22	1453	164	7	4.31
967	128	11	4.12	1213	147	9	4.23	1459	164	7	4.30
971	128	12	4.11	1217	147	9	4.22	1471	164	8	4.28
977	129	11	4.13	1223	147	10	4.21	1481	164	9	4.27
983	129	12	4.12	1229	148	9	4.23	1483	165	8	4.29
991	130	11	4.13	1231	148	9	4.22	1487	166	7	4.31
997	130	12	4.12	1237	148	10	4.21	1489	166	7	4.31
1009	132	10	4.16	1249	149	10	4.22	1493	166	7	4.30
1013	131	12	4.12	1259	150	9	4.23	1499	166	8	4.29
1019	132	11	4.14	1277	151	9	4.23	1511	166	8	4.28
1021	132	11	4.14	1279	151	9	4.23	1523	168	7	4.31
1024	124	20	3.88	1283	152	9	4.25	1531	169	7	4.32
1031	132	12	4.12	1289	152	9	4.24	1543	169	7	4.31
1033	133	11	4.14	1291	152	9	4.24	1549	170	7	4.32
1039	134	11	4.16	1297	153	9	4.25	1553	170	7	4.32
1049	134	11	4.14	1301	153	9	4.25	1559	170	7	4.31
1051	135	10	4.17	1303	153	9	4.24	1567	171	7	4.32
1061	135	11	4.15	1307	153	9	4.24	1571	171	7	4.32
1063	136	10	4.18	1319	154	9	4.25	1579	172	6	4.33
1069	136	11	4.16	1321	154	9	4.24	1583	172	7	4.33

Table 1 (continue). The smallest known sizes  $\bar{t}_2 = \bar{t}_2(2, q) < 4.5\sqrt{q}$  of complete arcs in planes  $PG(2, q)$ .  $A_q = \lfloor 4.5\sqrt{q} - \bar{t}_2(2, q) \rfloor$ ,  $B_q > \bar{t}_2(2, q)/\sqrt{q}$

$q$	$\bar{t}_2$	$A_q$	$B_q$	$q$	$\bar{t}_2$	$A_q$	$B_q$	$q$	$\bar{t}_2$	$A_q$	$B_q$
1597	173	6	4.33	1867	190	4	4.40	2129	206	1	4.47
1601	173	7	4.33	1871	191	3	4.42	2131	206	1	4.47
1607	174	6	4.35	1873	191	3	4.42	2137	206	2	4.46
1609	174	6	4.34	1877	191	3	4.41	2141	206	2	4.46
1613	174	6	4.34	1879	191	4	4.41	2143	207	1	4.48
1619	174	7	4.33	1889	191	4	4.40	2153	207	1	4.47
1621	174	7	4.33	1901	191	5	4.39	2161	207	2	4.46
1627	175	6	4.34	1907	192	4	4.40	2179	209	1	4.48
1637	176	6	4.35	1913	192	4	4.39	2187	209	1	4.47
1657	177	6	4.35	1931	194	3	4.42	2197	208	2	4.44
1663	177	6	4.35	1933	194	3	4.42	2203	209	2	4.46
1667	177	6	4.34	1949	194	4	4.40	2207	210	1	4.48
1669	177	6	4.34	1951	195	3	4.42	2209	210	1	4.47
1681	160	24	3.91	1973	196	3	4.42	2213	210	1	4.47
1693	179	6	4.36	1979	196	4	4.41	2221	210	2	4.46
1697	180	5	4.37	1987	197	3	4.42	2237	211	1	4.47
1699	179	6	4.35	1993	196	4	4.40	2239	211	1	4.46
1709	180	6	4.36	1997	198	3	4.44	2243	211	2	4.46
1721	181	5	4.37	1999	198	3	4.43	2251	212	1	4.47
1723	181	5	4.37	2003	198	3	4.43	2267	213	1	4.48
1733	182	5	4.38	2011	199	2	4.44	2269	213	1	4.48
1741	182	5	4.37	2017	199	3	4.44	2273	214	0	4.49
1747	182	6	4.36	2027	199	3	4.43	2281	214	0	4.49
1753	183	5	4.38	2029	200	2	4.45	2287	215	0	4.50
1759	183	5	4.37	2039	201	2	4.46	2293	215	0	4.49
1777	184	5	4.37	2048	201	2	4.45	2297	215	0	4.49
1783	183	7	4.34	2053	201	2	4.44	2309	215	1	4.48
1787	185	5	4.38	2063	202	2	4.45	2311	216	0	4.50
1789	185	5	4.38	2069	202	2	4.45	2333	217	0	4.50
1801	186	4	4.39	2081	203	2	4.45	2339	217	0	4.49
1811	187	4	4.40	2083	203	2	4.45	2341	217	0	4.49
1823	187	5	4.38	2087	203	2	4.45	2347	218	0	4.50
1831	188	4	4.40	2089	203	2	4.45	2351	218	0	4.50
1847	189	4	4.40	2099	204	2	4.46	2357	218	0	4.50
1849	189	4	4.40	2111	205	1	4.47	2371	218	1	4.48
1861	190	4	4.41	2113	205	1	4.46	2377	219	0	4.50

Table 2. The smallest known sizes  $\bar{t}_2 = \bar{t}_2(2, q) < 5\sqrt{q}$  of complete arcs in planes  $PG(2, q)$ .  $A_q = \lfloor 4.5\sqrt{q} - \bar{t}_2(2, q) \rfloor$ ,  $B_q > \bar{t}_2(2, q)/\sqrt{q}$ ,  $C_q = \lfloor 5\sqrt{q} - \bar{t}_2(2, q) \rfloor$

$q$	$\bar{t}_2$	$A_q$	$C_q$	$B_q$	$q$	$\bar{t}_2$	$C_q$	$B_q$	$q$	$\bar{t}_2$	$C_q$	$B_q$
2381	220		23	4.51	2551	229	23	4.54	2713	237	23	4.56
2383	220		24	4.51	2557	229	23	4.53	2719	238	22	4.57
2389	220		24	4.51	2579	230	23	4.53	2729	238	23	4.56
2393	221		23	4.52	2591	231	23	4.54	2731	238	23	4.56
2399	221		23	4.52	2593	231	23	4.54	2741	239	22	4.57
2401	192	28	53	3.92	2609	232	23	4.55	2749	239	23	4.56
2411	221		24	4.51	2617	233	22	4.56	2753	239	23	4.56
2417	221	0	24	4.50	2621	233	22	4.56	2767	241	22	4.59
2423	222		24	4.51	2633	232	24	4.53	2777	241	22	4.58
2437	222	0	24	4.50	2647	234	23	4.55	2789	241	23	4.57
2441	223		24	4.52	2657	233	24	4.53	2791	242	22	4.59
2447	223		24	4.51	2659	233	24	4.52	2797	241	23	4.56
2459	224		23	4.52	2663	235	23	4.56	2801	242	22	4.58
2467	224		24	4.51	2671	236	22	4.57	2803	242	22	4.58
2473	225		23	4.53	2677	236	22	4.57	2809	242	23	4.57
2477	225		23	4.53	2683	236	22	4.56	2819	242	23	4.56
2503	227		23	4.54	2687	236	23	4.56	2833	243	23	4.57
2521	227		24	4.53	2689	236	23	4.56	2837	244	22	4.59
2531	227		24	4.52	2693	237	22	4.57	2843	244	22	4.58
2539	228		23	4.53	2699	237	22	4.57	2851	244	22	4.57
2543	228		24	4.53	2707	237	23	4.56	2857	245	22	4.59
2549	229		23	4.54	2711	237	23	4.56	2861	245	22	4.59
									2879	245	23	4.57

$q$ , it is proved that  $t(\mathcal{P}_q) \leq \sqrt{q} \log^c q$ ,  $c = 300$ . The logarithm basis is not noted as the estimate is asymptotic. For definiteness, we use the binary logarithms. We introduce  $D_q(c)$  and  $\overline{D}_q(c)$  as follows:

$$t_2(2, q) = D_q(c) \sqrt{q} \log_2^c q, \quad \bar{t}_2(2, q) = \overline{D}_q(c) \sqrt{q} \log_2^c q.$$

From Tables 1, 2 and [2, Tab. 1], we obtain Observation 1.

**Observation 1.** Let  $173 \leq q \leq 2879$ ,  $q \neq 5^4, 3^6, 29^2, 31^2, 2^{10}, 37^2, 41^2, 7^4$ .

Then

(i)  $0.45 > \overline{D}_q(1) > 0.397$ . Also,  $0.428 > \overline{D}_q(1)$  if  $467 \leq q$ ;  $0.415 > \overline{D}_q(1)$  if  $1013 \leq q$ ;  $0.41 > \overline{D}_q(1)$  if  $1399 \leq q$ ;  $0.405 > \overline{D}_q(1)$  if  $1889 \leq q$ . So,  $\overline{D}_q(1)$  has a tendency to decreasing.

(ii)  $1.202 < \overline{D}_q(\frac{1}{2}) < 1.355$ . Also,  $\overline{D}_q(\frac{1}{2}) < 1.27$  if  $q \leq 443$ ;  $\overline{D}_q(\frac{1}{2}) < 1.32$  if  $q \leq 1291$ ;  $\overline{D}_q(\frac{1}{2}) < 1.325$  if  $q \leq 1327$ ;  $\overline{D}_q(\frac{1}{2}) < 1.335$  if  $q \leq 1801$ . So,  $\overline{D}_q(\frac{1}{2})$  has

a tendency to increasing.

(iii)  $0.720 < \overline{D}_q(0.75) < 0.743$ . The values of  $\overline{D}_q(0.75)$  oscillate about the average value 0.73331. It holds that

$$\begin{aligned} 0.720 < \overline{D}_q(0.75) < 0.743 & \quad \text{if } 173 \leq q \leq 997, \\ 0.727 < \overline{D}_q(0.75) < 0.741 & \quad \text{if } 1009 \leq q \leq 1999, \\ 0.729 < \overline{D}_q(0.75) < 0.738 & \quad \text{if } 2003 \leq q \leq 2879. \end{aligned} \quad (1)$$

Moreover, let

$$\widehat{t}_2(2, q) = 0.73331\sqrt{q}\log_2^{0.75} q, \quad \overline{\Delta}_q = \bar{t}_2(2, q) - \widehat{t}_2(2, q), \quad \overline{P}_q = \frac{100\overline{\Delta}_q}{\bar{t}_2(2, q)}\%.$$

It holds that

$$-1.86 \leq \overline{\Delta}_q \leq 1.23. \quad (2)$$

$$\begin{aligned} -1.73\% < \overline{P}_q < 1.31\% & \quad \text{if } 173 \leq q \leq 997, \\ -0.80\% < \overline{P}_q < 0.93\% & \quad \text{if } 1009 \leq q \leq 1999, \\ -0.53\% < \overline{P}_q < 0.54\% & \quad \text{if } 2003 \leq q \leq 2879. \end{aligned} \quad (3)$$

In other words,  $\widehat{t}_2(2, q) = 0.73331\sqrt{q}\log_2^{0.75} q$  can be treated as a **predicted** value of  $t_2(2, q)$ . Then  $\overline{\Delta}_q$  is the difference between the smallest known size  $\bar{t}_2(2, q)$  of complete arcs and the predicted value. Finally,  $\overline{P}_q$  is this difference in percentage terms of the smallest known size.

By (2),(3), the magnitude of the difference  $\overline{\Delta}_q$  is smaller than two. The magnitude of the percentage value  $\overline{P}_q$  is smaller than two for  $q < 1000$  and smaller than one for  $q > 1000$ . The region of  $\overline{P}_q$  is decreasing with growth of  $q$ . Also, by (1), the region of  $\overline{D}_q(0.75)$  is decreasing with growth of  $q$ .

The graphs of values of  $\overline{D}_q(0.75)$ ,  $\overline{\Delta}_q$ , and  $\overline{P}_q$  are shown on Figures 1-3.

Examples for great  $q$  are given in Table 3.

Table 3. The smallest known sizes  $\bar{t}_2 = \bar{t}_2(2, q) < 5\sqrt{q}$  of complete arcs in planes  $PG(2, q)$  with great  $q$ .  $B_q > \bar{t}_2(2, q)/\sqrt{q}$ ,  $C_q = \lfloor 5\sqrt{q} - \bar{t}_2(2, q) \rfloor$

$q$	$\bar{t}_2$	$C_q$	$B_q$	$\overline{D}_q(1)$	$\overline{D}_q(\frac{1}{2})$	$\overline{D}_q(\frac{3}{4})$	$q$	$\bar{t}_2$	$C_q$	$B_q$	$\overline{D}_q(1)$	$\overline{D}_q(\frac{1}{2})$	$\overline{D}_q(\frac{3}{4})$
3511	278	18	4.70	0.398	1.367	0.7380	5347	353	12	4.83	0.390	1.372	0.7312
4096	302	18	4.72	0.393	1.362	0.7319	5641	364	11	4.85	0.389	1.373	0.7307
4523	322	14	4.79	0.394	1.374	0.7360	5843	373	9	4.88	0.390	1.379	0.7335
5003	341	12	4.83	0.392	1.375	0.7345	6011	377	10	4.87	0.387	1.372	0.7291

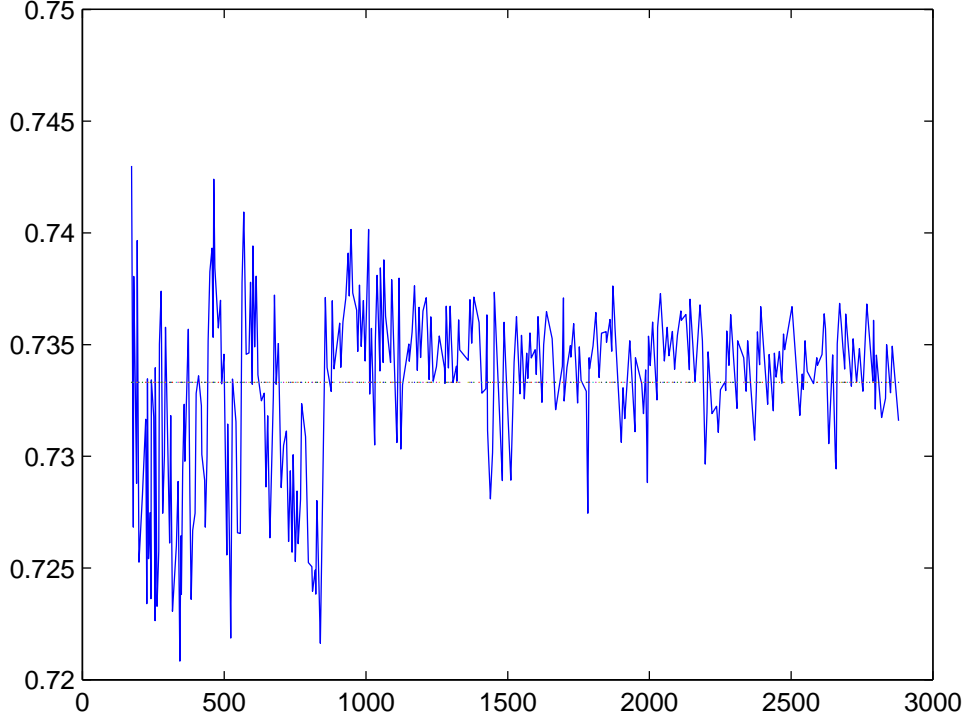


Figure 1: The values of  $\overline{D}_q(0.75) = \frac{\bar{t}_2(2,q)}{\sqrt{q} \log_2^{0.75} q}$ ,  $173 \leq q \leq 2879$ ,  $q \neq 5^4, 3^6, 29^2, 31^2, 2^{10}, 37^2, 41^2, 7^4$

The examples confirm Observation 1. So, along with  $B_q$ , the values  $\overline{D}_q(c)$ , in particular with  $c = 0.75$ , can be useful for estimates of complete arcs sizes.

Note that a complete 302-arc of Table 3 improves the result of [3] for  $q = 2^{12}$ .

From Tables 1-3 and [2, Tab. 1], we obtain Theorem 3.

**Theorem 3.** *Let  $173 \leq q \leq 2879$  and  $q = 3511, 4096, 4523, 5003, 5347, 5641, 5843, 6011$ . Then*

$$t_2(2, q) < 0.743\sqrt{q} \log_2^{0.75} q.$$

Taking into account (1) and Table 3, we assume that the following upper bound on the smallest size  $t_2(2, q)$  of complete arc in the plane  $PG(2, q)$  holds.

**Conjecture 1.** *It holds that*

$$t_2(2, q) < 0.75\sqrt{q} \log_2^{0.75} q, \quad 173 \leq q.$$

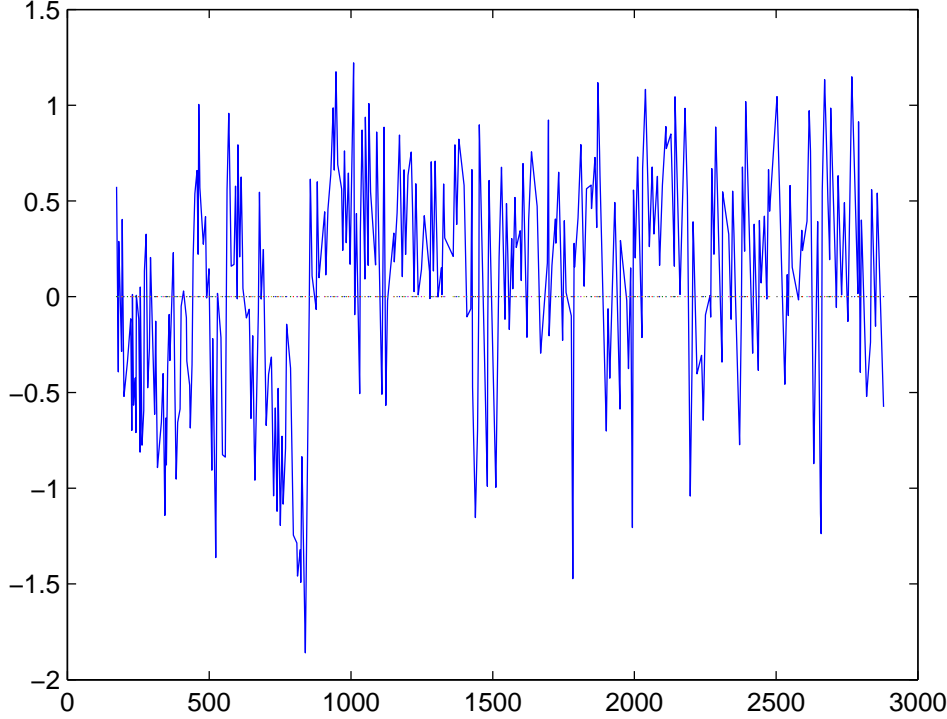


Figure 2: The values of  $\bar{\Delta}_q = \bar{t}_2(2, q) - 0.73331\sqrt{q}\log_2^{0.75} q$ ,  $173 \leq q \leq 2879$ ,  $q \neq 5^4, 3^6, 29^2, 31^2, 2^{10}, 37^2, 41^2, 7^4$

### 3 On the spectrum of possible sizes of complete arcs in $PG(2, q)$

Let  $m_2(2, q)$  be the greatest size of complete arcs in  $PG(2, q)$ . For odd  $q$ ,  $m_2(2, q) = q + 1$ . For even  $q$ ,  $m_2(2, q) = q + 2$ . For  $q = p^2$  there is the complete  $(q - \sqrt{q} + 1)$ -arc [10]. For  $q$  odd there is a complete  $\frac{1}{2}(q + 5)$ -arc [13]. For  $q \equiv 2 \pmod{3}$  odd,  $11 \leq q \leq 3701$  [2], and for  $q \equiv 1 \pmod{4}$ ,  $q \leq 337$  [6], there is a complete  $\frac{1}{2}(q + 7)$ -arc. For even  $q \geq 8$  there is a complete  $\frac{1}{2}(q + 4)$ -arc [9].

For even  $q$ , let  $M_q = \frac{1}{2}(q + 4)$ . For odd  $q$ , let  $M_q = \frac{1}{2}(q + 7)$  if either  $q \equiv 2 \pmod{3}$ ,  $11 \leq q \leq 3701$ , or  $q \equiv 1 \pmod{4}$ ,  $q \leq 337$ . Else,  $M_q = \frac{1}{2}(q + 5)$ .

Below we suppose that  $\bar{t}_2(2, q)$  is given in [2, Tab. 1] for  $q \leq 841$ ,  $q \neq 343$ , and in Tables 1 and 2 of this paper for  $853 \leq q \leq 2879$ . Also, in this work we



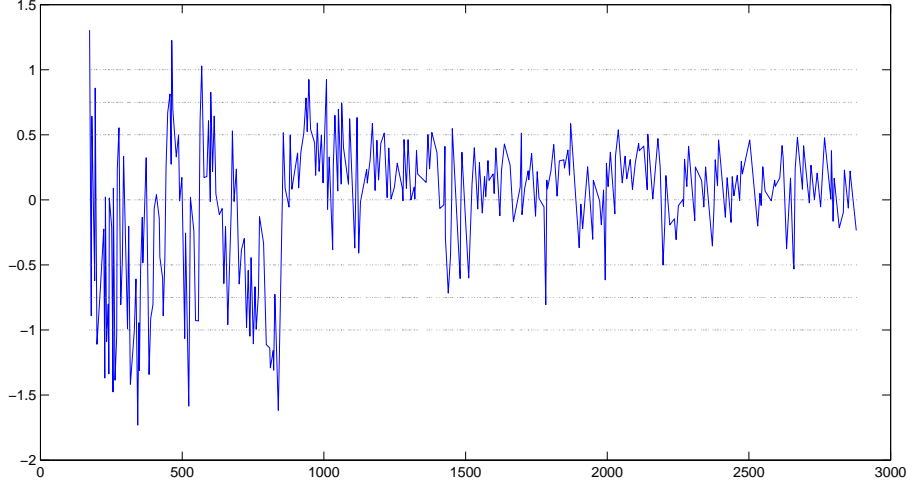


Figure 3: The values of  $\overline{P}_q = \frac{100\overline{\Delta}_q}{t_2(2,q)}\%$ ,  $173 \leq q \leq 2879$ ,  $q \neq 5^4, 3^6, 29^2, 31^2, 2^{10}, 37^2, 41^2, 7^4$

have obtained the value  $\bar{t}_2(2, 343) = 66$  that improves the result of [2].

**Theorem 4.** *In  $PG(2, q)$  with  $25 \leq q \leq 251$ ,  $257 \leq q \leq 349$ , and  $q = 1013, 2003$ , there are complete  $k$ -arcs of **all** the sizes in the region*

$$\bar{t}_2(2, q) \leq k \leq M_q.$$

*In  $PG(2, 256)$  there are complete  $k$ -arcs of sizes  $k = 55 - 123, 130, 241, 258$ .*

*Proof.* For  $25 \leq q \leq 167$  the assertion follows from [1, Tab. 2] and [2, Tab. 2]. For  $169 \leq q \leq 349$  and  $q = 1013, 2003$ , all the results are obtained in this work by the randomized greedy algorithms of [1, 5] with a new approach to creation of starting conditions and data.  $\square$

**Conjecture 2.** *Let  $353 \leq q \leq 2879$  be an odd prime. Then in  $PG(2, q)$  there are complete  $k$ -arcs of **all** the sizes in the region  $\bar{t}_2(2, q) \leq k \leq M_q$ . Moreover, complete  $k$ -arcs with  $\bar{t}_2(2, q) \leq k \leq \frac{1}{2}(q + 5)$  can be obtained by the randomized greedy algorithms of [1, 5] with a new approach to creation of starting data.*

Our methods are applicable using our present computers for  $q \leq 5171$ . For reason of space we plan to write more complete results and to describe this new approach to creation of starting conditions and data in a journal paper.

## References

- [1] A. A. Davydov, G. Faina, S. Marcugini, and F. Pambianco, Computer search in projective planes for the sizes of complete arcs, *J. Geom.*, **82**, 50–62, 2005.
- [2] A. A. Davydov, G. Faina, S. Marcugini, and F. Pambianco, On sizes of complete caps in projective spaces  $PG(n, q)$  and arcs in planes  $PG(2, q)$ , *J. Geom.*, **94**, 31–58, 2009.
- [3] A. A. Davydov, M. Giulietti, S. Marcugini, and F. Pambianco, New inductive constructions of complete caps in  $PG(N, q)$ ,  $q$  even, *J. Comb. Des.*, **18**, no. 3, 176–201, 2010.
- [4] A. A. Davydov, M. Giulietti, S. Marcugini, and F. Pambianco, On sharply transitive sets in  $PG(2, q)$ , *Innov. Incid. Geom.*, **6-7**, 139–151, 2009.
- [5] A. A. Davydov, S. Marcugini and F. Pambianco, Complete caps in projective spaces  $PG(n, q)$ , *J. Geom.*, **80** (2004) 23–30.
- [6] V. Giordano, Arcs in cyclic affine planes, *Innov. Incid. Geom.* **6-7**, 203–209, 2009.
- [7] M. Giulietti, Small complete caps in  $PG(2, q)$  for  $q$  an odd square, *J. Geom.*, **69**, 110–116, 2000.
- [8] A. Hartman and L. Raskin, Problems and algorithms for covering arrays, *Discrete Math.*, **284**, no. 1, 149–156, 2004.
- [9] J.W.P. Hirschfeld, *Projective geometries over finite fields*, Clarendon Press, Oxford, 1998.
- [10] J.W.P. Hirschfeld and L. Storme, The packing problem in statistics, coding theory and finite geometry: update 2001, in *Finite Geometries, Developments of Mathematics*, **3**, (Proc. of the Fourth Isle of Thorns Conf., Chelwood Gate, July 16-21, 2000), 201–246, Eds. A. Blokhuis, J.W.P. Hirschfeld, D. Jungnickel and J.A. Thas, Kluwer, 2001.
- [11] G. Keri, Types of superregular matrices and the number of  $n$ -arcs and complete  $n$ -arcs in  $PG(r, q)$ , *J. Comb. Des.*, **14**, 363–390, 2006.
- [12] J.H. Kim and V. Vu, Small complete arcs in projective planes, *Combinatorica*, **23**, 311–363, 2003.
- [13] G. Korchmáros and A. Sonnino, On arcs sharing the maximum number of points with an oval in a Desarguesian plane of odd order, *J. Comb. Des.*, **18**, 25–47, 2010.